CSU-TIPS RESEARCH LESSON STUDY PROPOSAL

2025-26

Adapted from CANMEE Lesson Study Proposal

For the lesson on: Relations, Equivalence classes
Place: Sonoma State (11-12-25), CSU Channel Islands (11-29-15?)
Instructor for Research Lesson: Ben & Jorge
Facilitator: Ben
Lesson Study Team Members: Ben, Jorge, Chad, Sayonita, William, Fred
Commentator: Aris Winter
Course: Intro to Proof

Title of Lesson: Properties of Relations: Introduction

Brief Description of Lesson: Using examples generated from their own birthdays and from game-play boards that they generate themselves, students will explore the characteristics of relationships that they identify. The lesson summary will use their examples to carefully define a relation on a set *A* and the properties *reflexive, symmetric, and transitive*.

1. **Students' competencies and challenges**

   Evidence / assets-based descriptors include: why each focal student was chosen; their funds of knowledge (cognitive and affective), content and agency strengths; and a content and agency goal for each focal student Add rows as needed.>

Ben's focal student interview video

Video from Jorge's Students Bennedy, Stephen and Benito

Focal Student	Competencies and Challenges
*** Teacher Grade	*Why chosen:* *Assets:* *Content Goal:* *Agency Goal:*
*** Teacher Grade	*Why chosen:* *Assets:* *Content Goal:* *Agency Goal:*
Student Name Teacher Grade	*Why chosen:* *Assets:* *Content Goal:* *Agency Goal:*

*** Teacher Ben Photo	*Why chosen:* Non-traditional student, prior trades career, *Assets:* Not willing to put up with half-understanding. Challenges me and other students when he doesn't understand an argument. Asks good questions *Content Goal:* Continue to grow in his understanding of math as a creative, discovery- and exploration-oriented activity *Agency Goal:* Share his own ideas, especially demonstrating physically with gestures or acting out.
*** Teacher Ben Photo	*Why chosen:* Quiet, not math major (liberal studies), not sure how deeply she's digging into thinking. Also not sure how she's affected by being one of only two women in the class. *Assets:* Willing participant/presenter. Starting to ask questions more. Open to feedback about reasoning *Content Goal:* Connect the definition of equivalence relation to examples from her own experience *Agency Goal:* Initiate conversations about her own ideas. Ask questions of other students to deepen her own understanding
*** Teacher Ben Photo	*Why chosen:* Sometimes has a deer-in-the-headlights look, but contributes readily. *Assets:* Happy to share ideas and thinking. Good partner in student teams. Starting to work with others outside of class. *Content Goal:* Be able to identify properties in everyday relations *Agency Goal:* Look to contribute his own ideas, rather than waiting to react to others
*** Teacher Ben No photo	*Why chosen:* Quiet, so I didn't have a good sense of how he was engaging *Assets:* Lately is pretty spot-on with his observations and suggestions. Thinks before speaking *Content Goal:* *Agency Goal:*

2. Rehumanizing STEM dimension and goal

<Resources: Standards for Mathematics Practices, Teaching for Robust Understanding, Aguirre et al 5 EQUITY PRACTICES The Impact of Identity in K-8 Mathematics Aguirre, Mayfield-Ingram, Martin; Bartell>

Rehumanizing STEM dimension: Body & Emotion; secondary: Ownership, Creation

Rehumanizing STEM Goal: Each student, especially focal students, will engage physically and emotionally with Proof-based concepts through hands-on, movement-based, or embodied activities that connect abstract ideas to lived experience.

3. Content goal

Each student, especially focal students will understand that some relations (equivalence relations) allow the set on which the relation is defined to be broken into subsets consisting of related elements (a partition), and others don't. They will understand and be able to recognize/check for some defining characteristics that contribute to the ability to form groups (reflexive, symmetric, transitive)

Each student, especially focal students will be able to

- explain the concept of equivalence relation by providing some examples where students are part of.

- Use their imagination to represent relations physically
- Determine if a given relation is transitive, reflexive or symmetric.

4. Connecting the Rehumanizing STEM dimension and the Content goal: How can the RS dimension support learning of the content?

5. Choose the task and anticipate student responses <Do the task and anticipate how focal students will do the task>

6. Unit plan < Describe lessons surrounding your research lesson, with a goal and task. Identify which lesson is the research lesson.>

Lesson	Unit Learning Goal and Tasks
1	Goal: see above Task: See lesson plan
2	Goal: Formalize definitions, use in proofs Task:
3	Goal: Task:
4	Goal: Task:
5	Goal: Task:

7. **Research Lesson Plan**

Indicated time percentages are recommendations; adjust as necessary

See notes at bottom for easier-to-follow plan

Steps, Learning Activities Teacher's Questions and Expected Student Reactions	Teacher Support	Formative Assessment
ACT 1 :: IGNITE AND LAUNCH (15%)		
Ignite<Connect to prior knowledge, experiences, funds of knowledge, context of focal students> **Wondering** <What question will students answer with this lesson, awaken their curiosity, motivate to engage?> Birthdays (William) ● Arrange yourself in a way that represents a way that your birthdays are related ● Now a different way ● Another ○ At least 3, one of which isn't equivalence	<Connect to funds of knowledge, competencies, context>	<Are students motivated to do the work?>
Launch Task < Describe the task as it will be presented to students. Give the exact phrasing of task with specific numbers used.> < How will students unpack the task so can access problem without being led to a strategy or solution> Sprouts game (Sayonita) ● Play the game ○ 4x6 cards ○ Draw big ○ Each on separate paper ○ Each player use different color ● Start with one dot ○ 2 possible games ● 2 dots ○ Get hang of game	< All students, especially focal students, access the task. What strategies support all students accessing the task? What prior knowledge will focal students bring to the task?>.	<Do focal students unpack the task?>

• Generate 10 graphs starting with 3 or 4 dots
  ○ Record: Winning player (#1 or #2)
  ○ Number of moves
  ○ Number of leftover vertices (degree < 3)
  ○ Enclosed regions produced
• Arrange by ways they're related
  ○ Pictures, then another way
  ○ Final: leave your trickiest secret relation arranged on table
• Groups challenge each other
  ○ Look at other groups' arrangements, try to identify relation
• In Bringing it All Together
  ○ Show pics of student arrangements
  ○ Compare/contrast to		

Steps, Learning Activities **Teacher's Questions and Expected Student Reactions**	**Teacher Support**	**Formative Assessment**
ACT II :: WORK TIME (50%)		
Workshop ≅20 mins <Students choose how they will work on the problem and what strategies to use. How will solo and group work build on anticipated focal student strengths? solutions?>	<Support students' multiple competencies>	
- **Anticipate student solution responses** <What are common issues / errors that may arise for this content?> < What are anticipated focal student solutions and representations and what questions will be asked to support students through productive struggle?>	<What teacher moves – model participation, accountable talk, position as capable and competent, use of representations and tools >	<Are these the anticipated responses? Do others emerge?>
- **Anticipate focal student exercise of agency** < How will encourage, recognize, and support focal student contributions to collective understanding of the content>	<What teaching practices might be used for each focal student? Other potential moves used to respond?>	<What are the effects of on focal students?>

- **Monitor and Sequence** <Monitor focal student work deciding who will share and in what order. How will you handle misconceptions during sharing? >	< Smith and Stein's note taking guide with anticipated solutions and teacher assessing and advancing questions>	< Are students explaining, justifying, questioning?>
Steps, Learning Activities **Teacher's Questions and Expected Student Reactions**	**Teacher Support**	**Formative Assessment**
ACT III :: BRINGING IT ALL TOGETHER (35%)		
Share and Connect ≅10 mins < What content do you want to elicit from student work, especially focal student's work? What content will you make sure is clear to students? How will you make connections between the different representations used?> Save representations of ● At least 2 equiv relations (groups) in two different contexts ● At least 2 non-equiv relations ○ One order relation (e.g. birthday earlier in year) ○ One non-transitive relation	● Assign competence ● Position students as source and expertise ● Distribute authority ● Validate knowledge, skills, strategies, dispositions ● Accountable to self and class ● Generate content questions ● Analyze, compare, justify ● Justify using evidence	<What connections do student make>
Summarize ≅5 mins <Build on focal student work to bring clarity to the content of the lesson. Use a concise summary statement.>		<Is the content clear? Concise? Connected to student work?>
Reflect ≅5 mins Student reflection prompt .. i.e., What did you learn from a friend today? What strategy will you try? What was the purpose of the lesson?		

8. "Board" plan < How will the board be designed to reveal the history of the lesson? Make visible student thinking? Help students make connections across representations? To make sense of the content? >

9. Observation question(s) <What question (about the lesson hypothesis) does the team want answered by the commentators and observers? Does the evidence from the points of assessment in the lesson plan lead to answering this question?>

10. **Reflections** < What was learned from the research lesson? About content? About access and agency? About the lesson hypothesis? About pedagogy? About focal students?>

    - Commentaries from public lesson (commentator)
      < synthesize findings from public lesson evidence and commentaries >

    - Team reflection < Professional learning of lesson study team >

    - Students—especially focal students'—reflections < Exit slips, surveys, meet and greet :: What was learned during this research lesson? How did you learn? In what ways was the lesson challenging?>

    - Administrator(s) reflection < What teacher and student learning was noticed? How does this inform site or district initiatives? >

Materials list:

- Blank paper
- 3x5 cards for birthdays
- Various colored markers to create game-play boards (so that pairs can have different colors)
- String and scissors in case students want to use

Launch: Relations and birthdays (15 min)

Each student choose "birthday" – can use real birthday or make one up

Each student write birthday and name on 3x5 card, large enough for others to see; also add # units

- What are some ways we might say we're "related" based on our birthdays?

First relation: student x is related to student y if x and y were born in the same month

- Students sort themselves into groups based on having the same birth month, ie. equivalence classes.
    - Take a picture
- What set A is our relation defined on?
- Are there any interesting properties you observe in this relation?
    - (maybe hint toward reflexive/symmetric/transitive but we won't introduce these until later; might not be much to work on with just one example)

Second relation: student x is related to student y if x and y's birthdays are within **60** days of one-another (ignoring year). (Number **60** should be adjusted based on class size to make interesting things happen)

- Students try to represent this relation by arranging themselves in the room
- What is different about this relation compared to the first?
- Goal: Somebody points out that we can make groups with first, not this one
- What interesting properties do you observe in a relation?
    - What's different between the two relations we've seen so far?

Third relation: student x is related to student y if x's birthday happens before or on the same day as y's birthday (again ignoring year).

- Students try to sort themselves into groups based on being related via this relation.
- What goes wrong this time? Is it different from last time?
- What interesting properties do you observe in a relation?

Bonus relation: we say that student x is related to the day of the year y if y is the birthday of x.

- What sets A and B is this a relation on now?
- What interesting properties do you observe in a relation?
- Another interesting type of relation is a function. Is this relation a function?

Workshop: Sprouts Game Challenge (35 min)

- Students in pairs (random, one trio OK. Ben uses https://www.randomlists.com/team-generator)
- Each game goes on its own sheet of paper. Draw big.
- Use different color pens for each player.
- Start with one dot on the paper. Do one & 2-dot examples on board to get rules
- Take turns connecting dots with curved lines, following these rules:
    - A line can connect a dot to itself or to another dot.
    - Lines cannot cross any other lines.
    - After every line, add a new dot somewhere along that line.
    - No dot can have more than three lines connected to it.
- When no more moves can be made, the game ends.
    - The last player to make a move wins.

Play several rounds:

- Start with one dot just to learn the rules.
- Then play two-dot games to get the hang of it.
- Finally, play with three dots and four dots.

Your challenge:

- Pre-printed 5 sheets per pair: 2 with 2 dots, 2 with 3 dots, 1 with 4 dots
- Generate 5 different Sprouts graphs (final pictures) starting with 2 or 3 dots.
- After a group has played 2, give them tally sheet
    - Winning player (first player or second)
    - Number of moves
    - Number of leftover vertices (dots with degree < 3)
    - Number of enclosed regions

When you are done:

~~a) Arrange your 10 graphs by how they are related: find patterns or categories.~~
b) Arrange your 10 graphs in a way that shows some comparison on connection
~~c) Make one arrangement by pictures, then make another arrangement based on a different idea.~~
~~d) Leave a relation displayed on your table without telling the rule that generated it. Don't make it too obvious, but not impossible either~~
~~e) Challenge another group to figure out your rule.~~

Final Step: As a class, we will look at several group arrangements and compare and contrast the ways you organized your Sprouts graphs.

Consolidation/Wrapup (25 min)

- Have student teams draw on board representations of four example relations (for Sprouts relations, teams generating relation will draw it)
    - If one of these examples can't be found among student teams, use the one in parentheses and either instructor write, or have a team write it up
    - Two Equivalence relations (One from bday (same month), one from Sprouts (total # dots))

    - Strict order (non-reflexive, non-symmetric, transitive) (Sprouts, A ~ B if A has fewer enclosed regions than B)
    - Non-transitive (use b'day within 60 days if no sprouts example available)
- Have each team explain their representation
- (2 minutes) teams try to name features of relations that allow them to be represented as discrete groups
    - Prompts?
- Define Relation R on a set A carefully: R \subseteq AxA
- Use these to define and name properties; in each case refer to examples on the board as examples or non-examples of the property
    - Symmetric: For every x, y \in A, x~y → y~x
    - Transitive: For every x,y,z \in A, (x~y and y~z) → x~z
    - Reflexive: For every x \in A, x~x
- Evaluate all relations on board for each of the relations
- Define Equivalence relation carefully

Assignment 17.12. Consider the two-player game **Sprouts**. Play begins with two dots on a blank page. On each turn, a player: draws a simple curve either between two existing dots, loops are allowed, and places exactly one new dot somewhere on that newly drawn curve. Constraints: the new curve may not cross any previously drawn curve, and each dot may be incident to at most three curves. The game ends when no legal move remains; the last player to move wins.

Let G_2 be the set of all graphs obtainable by legal play starting from the two initial dots. Define the following relations on G_2:

(a) Two graphs are related if they have the same number of dots at the end of the game.
(b) Two graphs are related if they have the same number of enclosed (bounded) regions in the plane.

Determine whether each of these relations is an equivalence relation on G_2.