Body & Emotion Lesson Study

Properties of Relations/
Equivalence Relations

Introduction to Proof

Sayonita Ghosh Hajra (Sacramento), William Casper (Fullerton) Jorge Garcia (Channel Islands), Chad Sprouse (Northridge), Ben Ford (Sonoma, Facilitator)

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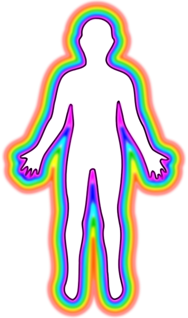
Project Team

Intro to Proof Lesson Study Campuses

Why Body and Emotions in a Proof Class?

- Traditional narrative: Mathematics is abstract and intellectual and does not have kinesthetic connections. All math is understood in one's mind.
- Counter-narrative
  - Emotions can give insight into the degree to which arguments are convincing
  - Physical referents (manipulatives) can help build mental models for ideas and lay down the structure for a written proof
  - Using "our" bodies to explore a concept makes you feel you are part of the understanding process, hence some ownership is attached to the concept.
  - "That is why we say that students first learn to move in new ways and then learn mathematics by modeling these new ways of moving" (Abrahamson & Sánchez-García, 2016).



Rehumanizing STEM goal

Each student, especially focal students, will engage physically and emotionally with Proof-based concepts through hands-on, movement-based, or embodied activities that connect abstract ideas to lived experience.

 Ryan Professor: Ben	 Nancy Professor: Ben	 Julio Professor: Ben
 Stephen Professor: Jorge	 Bennedy Professor: Jorge	 Benito Professor: Jorge

What are Relations?

Relations in mathematics are things like **>**, **<**, and **=** which capture relationships between different things.

- person A is related to person B if B is older than A
- number m is related to number n if m divides n

Relations that behave like the equals sign are called **equivalence relations**.

- animal A is related to animal B if they have the same species
- material X is related to material Y if they have equal tensile strength

Formally, they are **reflexive, symmetric, and transitive**

The Classes

Math 220 (Sonoma State): Reasoning and Proof (10 students)

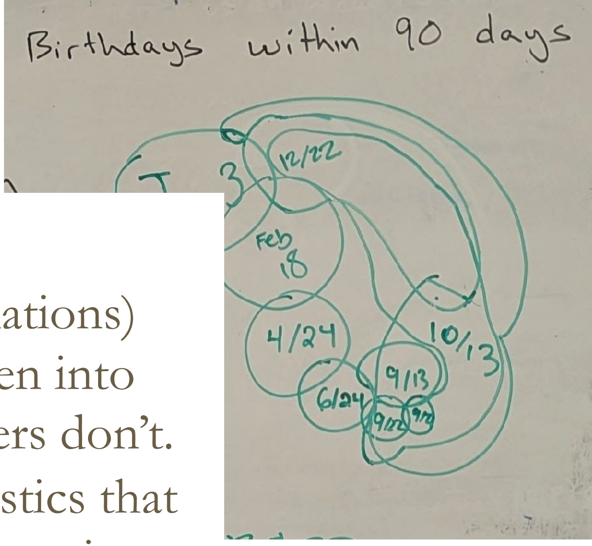
- Gateway to upper-division proof-based math courses
- Top-5 URM grade gap among math department courses across all years
- Transition from calculation-focused K-13 math work to logic of proof

Math 310 (Channel Islands): Transition into Higher Mathematics (12 students)

- Gateway to upper-division proof-based math courses
- Transfer students and non-transfer together in this class
- Students usually do not know sets/number theory/a bit insecure/shy
- Prerequisites: Logic and Calculus II

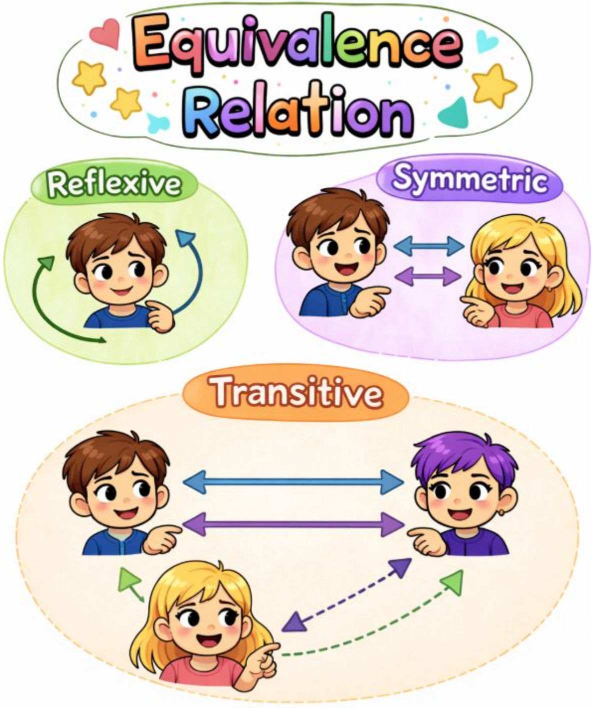
Content goal

- Each student, especially focal students will
  - Understand that whereas some relations (equivalence relations) allow the set on which the relation is defined to be broken into subsets consisting of related elements (a partition), others don't.
  - Be able to recognize/check for some defining characteristics that contribute to the ability to **form groups** (reflexive, symmetric, transitive relations)
  - Be able to explain the concept of equivalence relation by providing some examples where **students are part of**.
  - Use their imagination to represent relations **physically**.
  - Determine if a given relation is **transitive, reflexive or symmetric**.



Our process

- Through this activity we expected students to connect the concept of "relation" with themselves.
- Interview few students to plan the activity (focal students)
- Develop the plan for the activity
- Record lessons
- Students organize themselves into groups to represent relations
- Hoping students uncover the
  Three main properties of an equivalence relation
  - Transitive Peter~John, John~Mirna => Peter~Mirna
  - Symmetric Maria~Anton => Anton~Maria
  - Reflective AnyPerson~AnyPerson



Planning: How to “embody” thinking about relations?

Planning Discussion

- **Traditional approach:** Define properties (reflexive, symmetric, transitive) → then apply them to examples of relations.

- **Our approach:** Define and *act out* relations first → observe patterns → use those observations to describe properties (reflexive, symmetric, transitive).

Goal: Allow mathematical properties to emerge from embodied experience before introducing formal definitions.

Guiding Questions (Student Interviews)

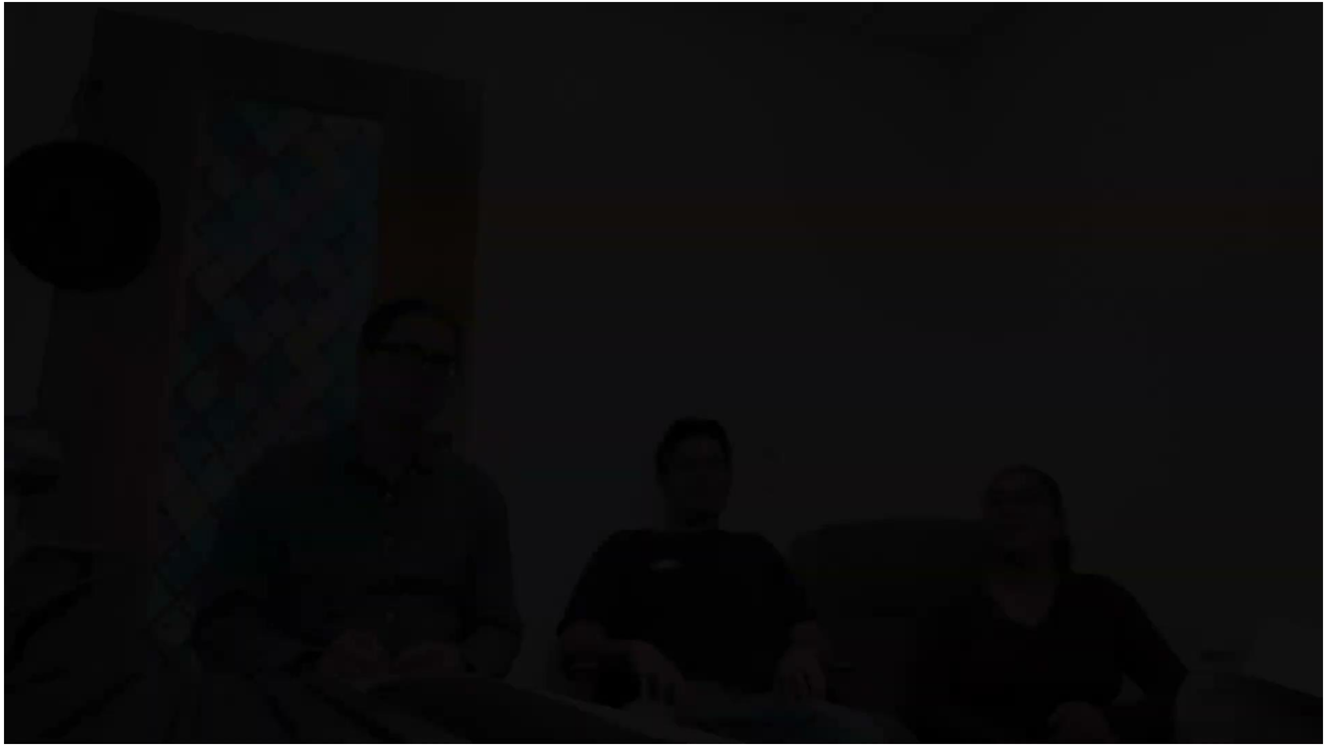
- Can you remember a math class where your personal experiences were used to explore a concept?

- Can you remember a math class where you moved or used your body to understand a math idea?

Choosing contexts

Relations defined by preferred foods, as a way to involve cultural connections.

Rejected due to lack of interesting responses during student interviews. Responses tend toward pizza/mexican/fast food but are generally noncommittal.



Choosing contexts

- Relations by grouping preferred foods, as a way to involve cultural connections. Rejected due to lack of interesting responses during student interviews. Responses tend toward pizza/mexican/fast food but are generally noncommittal
- Relations based on individual identity: Used birthdays as a rich source of possible groupings.
- Relation based on activities/games, to address body and emotion. First considered dice games, then settled on the game of sprouts.

Context #1: Birthdays

Using our birthdays (not year), how might we compare two people?

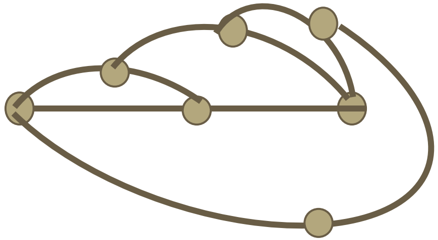
For example: “You and I share ____” or “My birthday is _________ yours”

- Birthday in same month
  - Or same day of month (not necessarily same month)
- Birthday within 60 days of each other
- Birthday person A earlier in year than person B
- Birthday person A earlier than or the same as person B

Now arrange in the room to represent one of these comparisons

Students rediscover “transitivity”

Context #2: Sprouts graphs



Two players start with several dots and take turns drawing a line between two dots (or a dot to itself), then placing a new dot on that line, splitting it into two.

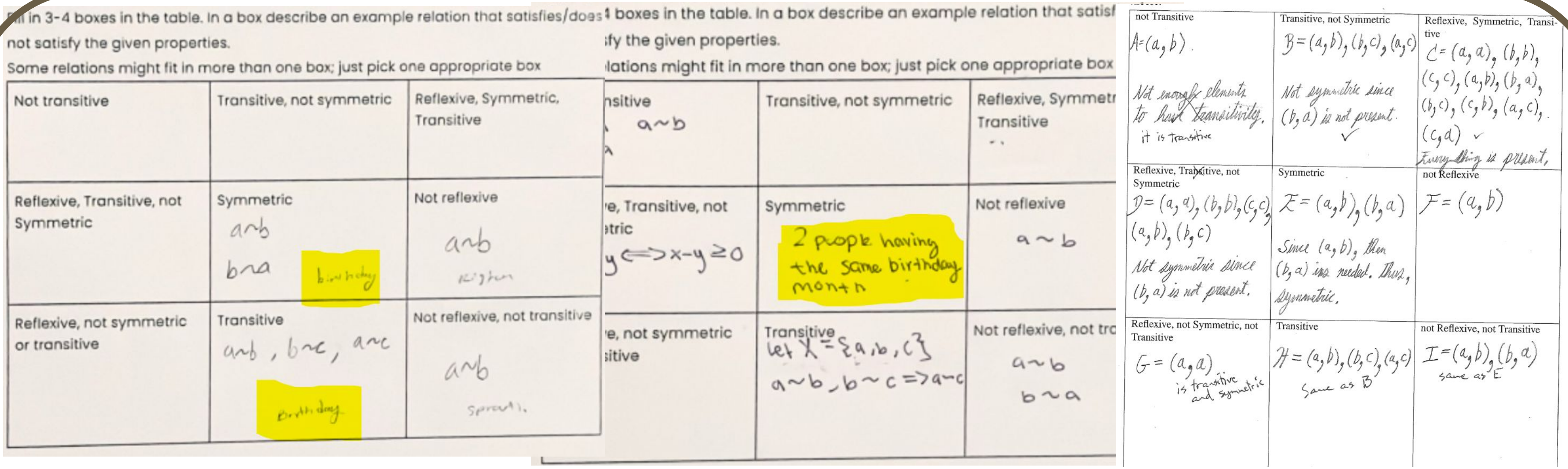
Rules:

- Lines cannot cross or touch any line, including themselves.
- The new dot cannot be placed on an endpoint.
- No dot may have more than three connections (a loop counts as two; a new dot has two).
- A line cannot touch the same dot more than once.

The player who places the last dot wins.

Evidence: Video

Evidence: Recall



Some students were able to retain the birthday-example of a relation we experienced by using our body to move around.

Debrief: Did we engage “Body and Emotion?”

Birthday context: The set that the relations were defined on was the people in the room: Definitely engaged bodies to represent various relations

Sprouts graphs: “Body” engagement felt forced and not a genuine “embodiment” of the target math

Conclusions

Not agreement in team about overall value of experience given the time required, and to what extent we addressed Body and Emotion.

If we were to do it again? Replace Sprouts, look for another characteristic to define relation on set of people in class

- Lots of things we have to avoid: personal characteristics, GPA. etc.
- Rethink food as a possible context
- Several members taking ideas into other classes, trying to incorporate more Body & Emotion activities

Conclusions

- How to hook into emotion?
  - Emotions experienced in previous 24 hours? Make list, related if shared.
  - Consider relation to Windows & Mirrors
  - Emotions about mathematics. Video:

Some Sources

- Liu, Zuo & Lu (2025) — *The Effect of Embodied Learning on Students' Learning Performance: A Meta-Analysis*
- Abrahamson & Bakker (2016) — Making Sense of Movement in Embodied Design for Mathematics Learning

Thank you!